SETS

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A set is a well defined collection of objects.

Definition — well defined ↓ we should be able to tell unambiguously whether some object belongs to a set or not.

e.g. Marie Curie is one of the Nobel laureates

$\therefore$ M. Curie $\in$ {Nobel laureates} → a set

but whether she was one of the five most influential scientists on earth? we can't say because everyone has their own opinion about that.

- Objects ≡ Elements ≡ Members of a set are all synonymous terms
- The set is denoted by capital letters & the elements are denoted by small letters

e.g. if an element a is a member of a set B we say $a \in B$ ($\in$: belongs to)

Examples:

N : the set of all Natural numbers
Z : the set of all integers
Q : the set of all rational numbers
R : the set of all real numbers
Z^+ : the set of all positive integers
C : the set of all complex numbers

Representation

To 'write' a set, we use two kinds of representations or forms:

(i) Roster / tabular form

A list of members, being separated by commas & enclosed within braces:

e.g.

The set of all natural numbers that divide 6 is written as:

$A = \{1, 2, 3, 6\}$ or $\{1, 3, 6, 2\}$ (order is not importa

(ii) Set-Builder form

Write the common property that each of the members share but is not shared by any object outside that set.

e.g.

① Set of all vowels of English alphabet:

$V = \{x : x$ is a vowel of English alphabet $\}$

V is equivalent to $V = \{a, e, i, o, u\} = \{e, i, o, u, a\}$

② Set of dividers (natural number) of 6:

$A = \{x : x$ is a number which divides 6 and $x \in Z^+\}$

Example:

#1 Write the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ in the set-builder form.

Ans: $\left\{x : x = \frac{n}{n+1}\right.$, where n is a natural number and $\left.1 \leq n \leq 6\right\}$

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#2 Write the set E = letters of the word TRIGONOMETRY in the roster form.

Ans $E = \{T, R, G, I, O, N, Y, E, M\}$

- order is not important
- repeated letters (i.e. elements) are written only once, in general.

Finite or Infinite?

If a set contains finite number of elements (including the case of no element at all), it is called a finite set. Otherwise it is an infinite set.

e.g.

A = { women employed in military } is a finite set

but B = { Set of all natural numbers greater than 45 } is an infinite set because there is an infinite no. of them greater than 45.

- To represent infinite set, we write 'dots', e.g.

$N = \{1, 2, 3 \ldots\}$

$Z = \{\ldots -1, 0, 1 \ldots\}$ etc.

* Not all infinite sets can be represented in roster form, because they may not share any common property. e.g. the set of all real numbers cannot be represented in roster form because they donot follow any pattern.

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Empty Set:

- A set which has no element at all.
- It is a type of finite set.
- Empty set is also called NULL SET or VOID SET.
- Denoted by ϕ.

Example: $A = \{x : x$ is a natural number and $x < 0\}$

$B = \{x : x$ is an even prime number and is greater than $2\}$

etc.

Equal sets:

Two sets A & B are equal iff (if and only if) given a member a of A, is also a member of b and vice-a-versa.

i.e. $a \in A \Rightarrow a \in B$
& $a \in B \Rightarrow a \in A$ $\Big\} \Rightarrow A = B$

$\Rightarrow$ (implies)

- Equal sets have exactly same elements.

- A set does not change if one or more elements are repeated.

e.g. ① $A = \{1, 2, 3, 4\}$
& $B = \{1, 2, 2, 2, 2, 3, 3, 4\}$ are equal.

② $A = \{x : x^2 = 9\}$ and $B = \{y : y - 3 = 0\}$
are NOT equal
because $A = \{3, -3\}$
but $B = \{3\}$.

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Subset

A set A is said to be a subset of a set B if every element of A is also an element of B.

i.e. if $a \in A \Rightarrow a \in B$

$\Rightarrow \underbrace{A \subseteq B}_{\text{subset of}}$

e.g. $A = \{1,2,3\}$; $B = \{1,2,3,4,5\}$ $\Rightarrow A \subset B$

(note that $B \not\subset A$, hence, $A \subset B \not\Rightarrow B \subset A$. If $A \subset B$ & $B \subset A$ $\Rightarrow A = B$)

- A set which has only one element is called a SINGLETON SET.

  e.g. $A = \{1\}$, $B = \{-45\}$, $C = \{\phi\}$ etc.

  (* ϕ is itself null set but can be an element of another set)

* ϕ is a subset of every set.
* An element of a set can never be a subset of itself.

e.g. if $A = \{3\}$

$B = \{\{3\}, 2, 4\}$

Then $A \not\subset B$ because $3 \in A$ but $3 \not\in B$

However, $A \in B$.

i.e. A is not a subset of B, but is an element of it.

Numbers: $N \subset Z \subset Q \subset R$

Intervals

$[a,b) \equiv \{x : a \leq x < b\}$

i.e. open interval from a to b, including a but excluding b.

$(a,b] = \{x : a < x \leq b\}$

i.e. open interval from a to b, including b but excluding a.

examples:

- $(-\infty, \infty) \rightarrow$ set of all real numbers
- if $A = (-2, 2)$
  & $B = [-2, 3)$
  Then $A \subset B$

* An interval contains infinitely many points.

e.g. 

$\equiv \quad (-2,2) = A$

Here, every point on the brown line belogs to A.

* $(b-a)$ is the length of any of the intervals
(a,b) or $[a,b]$ or $(a,b]$ or $[a,b)$.

POWER SET

The set of all subsets of a set A (including itself) is called the power set P(A).

* Every element of " " P(A) is a set.

e.g. if $B = \{3, 4, -5\}$
then $P(B) = \{\{3\}, \{4\}, \{-5\}, \phi, \{3,4\}, \{4,-5\}, \{3,-5\}, \{3,4,-5\}\}$

* no. of elements in $P(A) = 2^m$
if no. of elements in $A = m$

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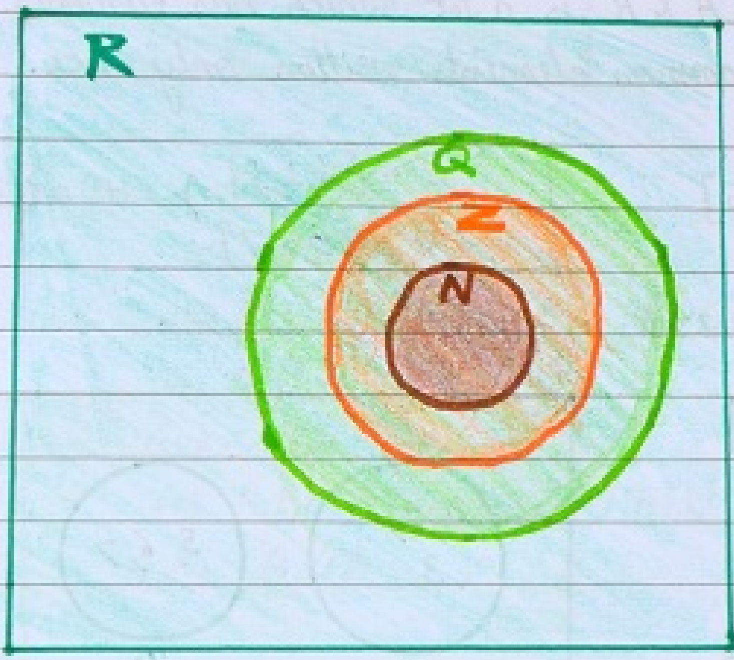
Universal Set

A basic set which is relevant to that particular content in which our problem lies. It includes every element of every set encountered in that context.

e.g. when dealing with natural numbers, the universal set is can be the set of all integers or, the set of rational numbers or, the set of real numbers etc.



N : Natural numbers
{1,2,3....}

Z : Integers
{...,-3,-2,-1,0,1,2,3...}

Q : Rational numbers
e.g. $\frac{7}{3}$, $-\frac{2}{4}$, 0, $\frac{5}{1}$ etc.

R : Real numbers → Universal Set
{-∞,∞}

$N \subset Z \subset Q \subset R$

Venn Diagrams

Depict relations between sets.

e.g. see above (numbers).

Universal set → ▭

Subsets → ○

elements can be written inside their respective set

overlapping areas → common elements